CRREL (12) LEVEL

Shallow snow model for predicting vehicle performance

AD A108343

DTIC
ELECTE
DEC 10 1981
S D
B

81 12 08 105

CRREL Report 81-20

Shallow snow model for predicting vehicle performance

William L. Harrison

October 1981

Prepared for
OFFICE OF THE CHIEF OF ENGINEERS
By
UNITED STATES ARMY CORPS OF ENGINEERS
COLD REGIONS RESEARCH AND ENGINEERING LABORATORY
HANOVER, NEW HAMPSHIRE, U.S.A.

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER CRREL Report 81-20	2. GOVT ACCESSION NO. AD-A108343	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) SHALLOW SNOW MODEL FOR PREDICTING VEHICLE PERFORMANCE		5. TYPE OF REPORT & PERIOD COVERED
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) William L. Harrison		8. CONTRACT OR GRANT NUMBER(s) 3c
9. PERFORMING ORGANIZATION NAME AND ADDRESS U.S. Army Cold Regions Research and Engineering Laboratory Hanover, New Hampshire 03755		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS DA Project 4A762730AT42 Task A, Work Unit 010
11. CONTROLLING OFFICE NAME AND ADDRESS Directorate of Military Programs Office, Chief of Engineers Washington, D.C. 20314		12. REPORT DATE October 1981
		13. NUMBER OF PAGES 25
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Mobility
Models
Shallow depth
Snow
Snow vehicles
Traction

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A historical review of research is presented to establish the state-of-the-art for analyzing the behavior of vehicles in shallow snow. From this review, the most comprehensive and promising model is put together to establish a first-cut performance prediction model for vehicles operating in shallow snow, slush, ice and thawing soils.

DD FORM 1473 1 JAN 73 EDITION OF 1 NOV 65 IS OBSOLETE

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

031 200

PREFACE

This report was prepared by Dr. William L. Harrison, Research Civil Engineer, of the Applied Research Branch, Experimental Engineering Division, U.S. Army Cold Regions Research and Engineering Laboratory. Funding for this research was provided by DA Project 4A762730AT42, *Design, Construction and Operations Technology for Cold Regions*, Task A, *Combat Operations Support*, Work Unit 010, *Battlefield Mobility in Cold Regions.*

The manuscript of this report was technically reviewed by Dr. R.A. Liston and G.L. Blaisdell of CRREL.

The contents of this report are not to be used for advertising or promotional purposes. Citation of brand names does not constitute an official endorsement or approval of the use of such commercial products.

CONTENTS

ILLUSTRATIONS

TABLES

LIST OF SYMBOLS

A	contact area (m^2)
A_p	peripheral area (m^2)
B_1	shear-stress/strain parameter (m^{-1})
b	track or wheel width (m)
C_c	vehicle characteristic constant
c	internal shearing resistance (cohesion) (kPa)
c_a	interface shearing resistance (adhesion) (kPa)
D	drawbar pull
d	tire diameter (m)
E	compaction energy (J)
F	vertical force (N)
f_i	coefficient of friction
g	acceleration of gravity (m/s^2)
H	gross tractive effort (N)
h	snow depth or layer thickness (m)
i	slip ratio
j	slip distance (m)
k	contact length (m)
ℓ	contact length (m)
n_a	number of axles on vehicle
n_f	maximum number of tires per axle
p, p_z	contact pressure (kPa)
P_c	carcass stiffness (kPa)
p_i	tire inflation pressure (kPa)
P_z	pressure at sinkage z (kPa)
R	resistance to motion (N)
R_c	compaction resistance (N)
R_t	total resistance (N)
r	tire radius (m)
V	speed (m/s)
V_s	interface velocity (m/s)
V_1	initial volume (m^3)
V_2	final volume (m^3)
W	vehicle weight (kg)
z_1, z_e, z	sinkage (m)
α	vehicle trim (degrees)
β	plastic kinematic viscosity (m^2/s)
γ_1	bulk density or specific weight (Mg/m^3)
γ	critical density (Mg/m^3)
γ_i	initial density (Mg/m^3)
γ_0	compacted density (Mg/m^3)
δ	tire deflection (m)
θ	slope or grade (degrees)
μ	tan ϕ
ρ	angle of approach (degrees)
σ	normal stress (kPa)
σ_t	tensile strength of ice (kPa)
τ	shearing strength (kPa)
τ_ϕ	dynamic shearing resistance (kPa)
ϕ	angle of internal shearing resistance (degrees)
ω	work of compaction per unit volume (J/m^3)
Φ	angle of interface shearing resistance (degrees)

SHALLOW SNOW MODEL FOR PREDICTING VEHICLE PERFORMANCE

William L. Harrison

INTRODUCTION

The state-of-the-art of predicting vehicle peformance in snow is still at the pioneering level. Although work has been done towards this end for at least 25 years, there has yet to appear in the literature a prediction model based on the treatment of snow as a unique material (Yong and Harrison 1978). It is anticipated that this situation will change in the next few years and attention will be given to the development of a proper snow failure model.

The purpose of this report is to review all past efforts that have been devoted to the prediction of vehicle performance in shallow snow layers and establish the current state-of-the-art. Within this context, a performance model will be recommended for use as a "best estimate" of the traction and resistance developed by wheeled and track-laying vehicles in shallow snow.

In this report, consideration will be given to the modification of traction and resistance forces by vehicle morphology and various types of traction aids. The snow cover considered will range in strength properties from those encountered in dry snow to those of slush layers, and the snow will be presumed to be deposited on a firm base. An assessment will be made of the accuracy of current methods and a model will be based on available test data from past exercises.

HISTORICAL REVIEW

The literature relating specifically to vehicle mobility through shallow snow (ground support) is not extensive. The problem has, however, been sparingly addressed in the last 25 years relative to vehicle performance predictions. It is generally agreed upon in the literature that the definition of shallow snow depends on the vehicle of interest. The snowpack is said to be shallow when the disturbed volume (pressure bulb) beneath a traction element is in contact with the ground.

Within this context of vehicle performance in shallow snow, several different layer conditions must be considered. The first condition, which represents the idealized representation of shallow snow, is that of a homogeneous layer of settled snow covering a nondeforming subsurface. The second condition is that of a packed snow layer over a nondeforming subsurface.

Other surface conditions such as glare ice, slush, snow-ice, and thawing soil are of interest in this study but will be addressed later in this report.

Nuttall (1957) described a possible approach to predicting the resistance force acting on vehicles traveling in shallow snow layers. It will become apparent that many subsequent efforts bear some degree of similarity to this proposition. Nuttall's basic premise was based on pressure-density data which in general appearance is as shown in Figure 1.

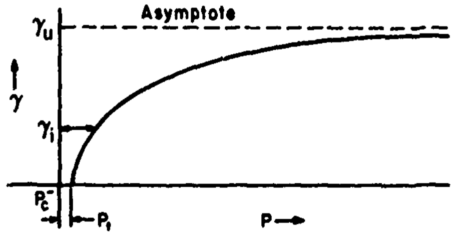


Figure 1. Pressure-density curves.

The relationships developed by Nuttall were an application of the Bernstein (1913) and Bekker (1952) equations for the work of compaction:

$$R = \int_0^z p\, dz \tag{1}$$

Nuttall expresses p in terms of density and obtains

$$\frac{R}{F} = \frac{h}{\ell} \frac{p_z}{p} \left(1 - \frac{\gamma_i}{\gamma_0}\right) \tag{2}$$

where R = snow resistance to motion (N)
F = vertical force (N)
R/F = dimensional quantity of resistance (N) per applied load (N)
h = snow depth (m)
ℓ = contact length (m)
p_z = pressure at sinkage z (kPa)
p = average ground pressure (kPa)
γ_i = initial snow density g/cm³
γ_c = final snow density g/cm³

If eq 2 were expressed in terms of resistance per unit change in volume and density in terms of sinkage, it would take the following form:

$$R = bp_z z \tag{3}$$

(since $p_z = F/b\ell$ and $z = h[1-(\gamma/\gamma_0)]$) where b is track or tire width and h snow depth. Equation 3 bears a close resemblance to the form postulated by Harrison (1975) which is discussed later.

Nuttall and McGowan (1961) collaborated on work using scale models of vehicles in snow to predict the performance of full-sized vehicles. The basic relationship developed and used to express performance was D/W vs the load numeric ($W/c\,d^2$), where D is drawbar pull, d tire diameter, W vehicle weight, and c the structural cohesion. This relationship comes from the dimensional equations of the dependent variables of sinkage z, vehicle trim a, and drawbar pull D.

A significant feature of the work by Nuttall and McGowan (1961) is that, when establishing dimensional equations, much thought was given to the factors relating to the vehicle, to the snow, and to the system as a whole. The following listing from Nuttall and McGowan (1961) is considered worthwhile repeating as most of these factors must be considered in this study. Some items have been updated or modified by more recent research efforts.

Factors relating to the vehicle

1. Geometry or/and configuration, i.e. length, wheel base, dimensional aspects of traction elements, and number of propulsion units (articulated configuration, etc.).
2. Weight.

Factors relating to the operation of the vehicles

3. Speed, including slip.
4. j = slip distance of a tractive element relative to the undisturbed material during one cycle in contact with the material.
5. V = a characteristic *speed* related to the vehicle, such as peripheral wheel speed relative to the vehicle. At a given moment, all other velocities and components in the system may be expressed in terms of ratios to this speed. One ratio of particular importance in dynamic systems is the slip ratio i:

$$i = V_s / V \tag{4}$$

where V_s is the slip speed or average speed of the wheel contact area relative to the undisturbed soil or snow. It is to be noted that, while the slip ratio is usually conceived in terms of speed, it also has a simple geometric interpretation:

$$i = j/\ell \tag{5}$$

where j is defined in factor 4 and ℓ is the distance from front to rear of the running gear contact area along the same path over which j is measured (usually parallel to the average surface of the material). In terms of speeds, the slip ratio implies dynamic snow reactions arising from inertial and/or viscous effects. In terms of distances, the ratio implies essentially static material reactions, determined primarily by displacement but little influenced by time-rate of displacement. (While both effects may be implied, dimensional analysis will not tell you which.)

Factors relating to the snowpack

6. h = total *depth* of material of interest. Boundaries of significant

layers, and hence thicknesses as well, may be specified by measurements from the surface.

7. c = before-collapse, or *structural cohesion*, of the material at depth h. This will, in general, vary at a given spot and time from layer to layer, and hence with depth.

8. τ_ϕ = the full, unit after-collapse, or *dynamic shearing resistance*, of the material originally at depth h. For any depth, it may be approximated by Coulomb's assumption:

$$\tau_\phi = c + \sigma \tan\phi \tag{6}$$

where σ is the unit normal loading on the shear plane. On this basis, τ_ϕ may be replaced by c and $\tan\phi$, where these are, respectively, the effective cohesion and the tangent of the effective angle of internal friction of the collapsed material originally at depth h.

9. γ = the before-collapse bulk specific weight of the material at depth h.

10. β = kinematic viscosity at depth h.

11. $\tan\theta$ = slope of the surface of the material.

12. f_i = coefficient of friction of soils or snows at depth h in the material.

13. B = stress–strain parameter characterizing dynamic shearing resistance in the layer. Dynamic shearing resistance develops only after some consolidation and reorientation of the grains of the collapsed material. The typical relationship in weak, loose materials between unit shearing resistance and shear travel may be approximated by

$$\tau = \tau_\phi \ (1 - e^{-Bj}) \tag{7}$$

Because Bj must be dimensionless, the dimension of the parameter B is ℓ^{-1}.

The list might be further lengthened by the inclusion of soil or snow elastic properties, but the phenomena of interest regularly involve large, permanent deformations of the material, so that elastic forces may be neglected from the outset, at least until experimental evidence of their importance dictates otherwise.

Factors relating to the system as a whole

14. g = acceleration of gravity.

Dependent variables

15. z = sinkage of vehicle.

16. α = trim of vehicle

17. D = drawbar pull, or measurable margin of tractive capacity over external motion resistance. Drawbar pull of a given machine in a given material is largely influenced by slip ratio i and/or grouser travel j. Either drawbar pull or slippage may be taken as the independent variable, and the remaining one as dependent. In testing, it is convenient to control slippage and to measure drawbar output. In practice, the drawbar load is fixed, at any given moment, by terrain and towed load, and slippage becomes the dependent variable.

Most of these factors will appear in the algorithm for computing gross tractive effort and snow resistance to motion.

Blackmon and Rula (1960) presented a compilation of tests conducted in Boulder and Camp Hale, Colorado; Fort Churchill and Kapuskasing, Canada; and Houghton, Michigan. Some of the tests can be considered as shallow snow tests. Figure 2 shows snow depth vs rut depth of two Army vehicles operating at Camp Hale, Colorado: a M8E2 cargo tractor and a M5A4 high-speed tractor.

Plots of the performance of the M8E2 and the M5A4 (ground pressure ≈ 42.7 and 57.9 kPa, respectively) indicate that the compacted snow density beneath the ruts remained the same regardless of the original snow depth; i.e. rut depth = snow depth x (γ/γ'). Assuming that the virgin snow density γ was constant, the compacted snow density γ' would also have to be constant in order that the slope of the curves in Figure 2 remain constant. Figure 3 (again from Blackmon and Rula 1960) shows the relationship between maximum drawbar-pull and snow depth for the M8E2 and the M5A4.

Figures 2 and 3 indirectly show the relationship between rut depth and resistance in shallow snow for the M8E2 and M5A4. Figure 4 (from Abele and Parrott 1968) gives a tractive coefficient for the same two vehicles of $\tan\phi = 0.44$.

The tests conducted at Houghton, Michigan, by Abele and Parrott (1968) did not include the same type of data as did tests at Camp Hale, and no assessment could be made of the two vehicles most likely to give shallow snow test results: namely the D4 and D7 tractors in approximately 46-cm snow depth. One very interesting result which comes from Table 24 of Abele and Parrott (1968) (a

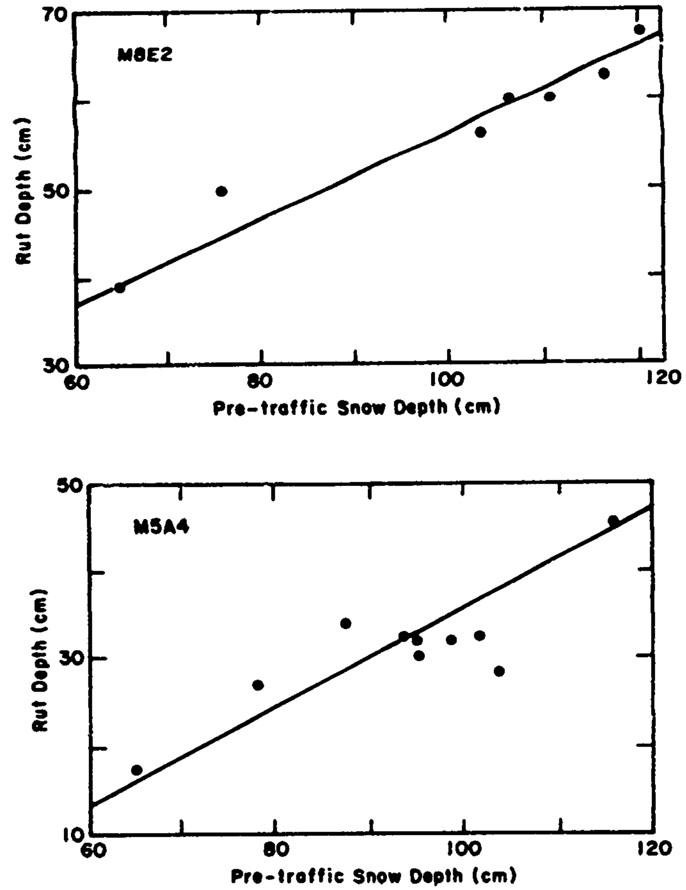


Figure 2. Self-propelled tests: snow depth vs rut depth, first test period, Camp Hale, Colorado (from Blackmon and Rula 1960).

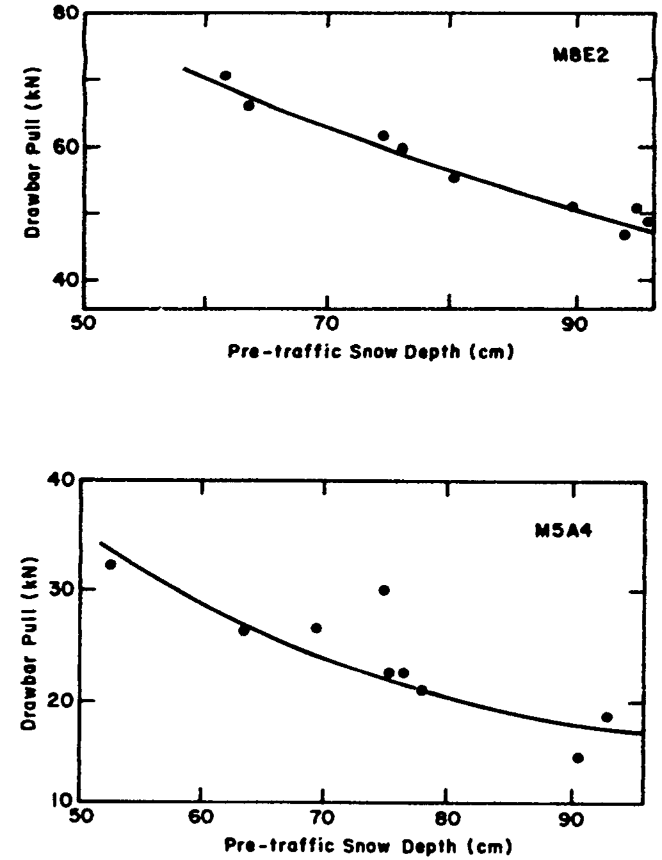


Figure 3. Snow depth vs maximum drawbar-pull: first test period, Camp Hale (from Bla^kmon and Rula 1960)

Table 1. Shallow snow data—snow characteristics.

Location	T* (°C)	γ (g/cm^3)	h (cm)	z (cm)	d† (cm)	ω (Joules/cm^3)	cu (kPa)	Φ (degrees)	c (kPa)	ϕ (degrees)
Houghton	−7	0.15	14	10	20.3	0.30	0	18.8	1.5	18.2
30 Jan 75	−7	0.15	16	12	20.3	0.50	0	18.8	1.5	18.2
	−7	0.15	14	11	20.3	0.35	0	18.8	1.5	18.2
	−7	0.15	11	9	20.3	0.29	0	18.8	1.5	18.2
	−7	0.15	12	10	20.3	0.42	0	18.8	1.5	18.2
	−7	0.15	14	10	25.4	0.29	0	18.8	1.5	18.2
	−7	0.15	17	13	20.3	0.36	0	18.8	1.1	20.2
	−7	0.15	18	14	20.3	0.64	0	18.8	1.1	20.2
	−7	0.15	19	15	20.3	0.55	0	18.8	1.1	20.2
	−7	0.15	18	14	25.4	0.59	0.27	17.5	1.1	20.2
	−7	0.15	21	15	20.3	0.49	0.27	17.5	0.89	20.2
	−7	0.15	21	16	20.3	0.63	0.27	17.5	0.89	20.2
	−7	0.15	20	16	20.3	0.66	0.27	17.5	0.89	20.2
	−7	0.15	19	13	25.4	0.42	0	20.3	0.89	20.2
	−5	0.14	17	12	20.3	0.55	0	20.3	0.4	22.2
	−5	0.14	16	12	20.3	0.50	0	20.3	0.4	22.2
	−5	0.14	16	11	20.3	0.36	0	20.3	0.4	22.2
	−5	0.14	13	10	20.3	0.48	0	20.3	0.4	22.2
	−5	0.14	14	11	20.3	0.57	0	20.3	0.4	22.2
	−5	0.14	16	12	20.3	0.58	0	20.3	0.4	22.2
	−5	0.14	14	10	25.4	0.31	0	20.3	0.45	22.2
19 Feb 75	−12	0.26	11	7	20.3	0.60	0	22.8	0.13	25.9
	−12	0.26	11	8	20.3	0.91	0	22.8	0.13	25.9
	−12	0.26	11	8	20.3	0.60	0	22.8	0.13	25.9
	−12	0.26	10	7	20.3	0.70	0	22.8	0.13	25.9
	−12	0.26	11	6	25.4	0.40	0	22.8	0.13	25.9
	−12	0.26	12	6	25.4	0.60	0	22.8	0.13	25.9
	−4.5	0.23	14	9	20.3	0.28	0.34	25.2	0.69	24.7
	−4.5	0.23	11	9	20.3	0.58	0.34	25.2	0.69	24.7
	−4.5	0.23	12	5	20.3	0.38	0.34	25.2	0.69	24.7
	−4.5	0.23	9	6	25.4	0.26	0.34	25.2	0.69	24.7
3 Mar 75	−3	0.15	9	7	20.3	0.17	0.27	22.8		
	−3	0.15	10	7	20.3	0.28	0.27	22.8		
	−3	0.15	12	8	20.3	0.27	0.27	22.8		
	−3	0.15	7	6	25.4	0.05	0.27	19.4		
	−3		7	6	20.3	0.14	0.27	19.4		
	−3		8	6	20.3	0.21	0.27	19.4		
	−3	0.30	12	7	20.3	0.38	0	19.8		
	−3	0.30	12	9	20.3	0.50	0	19.8		
	−3	0.30	13	0	20.3	0.32	0	19.8		
	−3	0.30	16	10	25.4	0.35	0	19.8		
4 Feb 76			11	8	15.2	1.27	1.0	21		
			10	7	15.2	1.01	1.0	20		
			10	8	15.2	1.42	1.0	18		
			8	7	15.2	1.34	1.0	19		
5 Feb 76			9	7.7	15.2	1.61	1.0	21		
			11	9.3	15.2	1.24	1.4	21		
18 Feb 76			15	10	15.2	0.69	1.2	33		
			14	11	15.2	0.92	1.2	33		
			17	12	15.2	0.80	1.2	33		
			18	12	15.2	0.75	1.2	33		
19 Feb 76			16	13	15.2	1.33	0.68	24.7		
			16	13	15.2	1.32	0.68	24.7		
			15	12	15.2	1.39	0.68	24.7		

* Snow temperature.

† Plate diameter.

Table 1 (cont'd). Shallow snow data—snow characteristics.

Location	T (°C)	γ (g/cm³)	h (cm)	z (cm)	d (cm)	ω (joules/cm³)	ca (kPa)	Φ (degrees)	c (kPa)	φ (degrees)
Silver Creek Bridge*	- 5	0.1	29.2	17	5.0	14.4†			0.44	20.8
30 Jan 73	- 5	0.1	29.2	15	7.6	9.00			0.44	20.8
	- 5	0.1	29.2	13	10.2	5.03			0.44	20.8
	- 5	0.1	29.2	9	12.7	2.80			0.44	20.8
	- 5	0.1	29.2	16	7.6	9.10			0.02	31.2
	- 5	0.1	29.2	15	10.2	5.65			0.02	31.2
	- 5	0.1	29.2	12	12.7	3.4			0.02	31.2
	- 5	0.12	25.4	15	7.6	10.33			0	27
	- 5	0.12	25.4	11	10.2	4.59			0	27
	- 5	0.12	25.4	9	12.7	2.30			0	27
	- 6	0.12	25.4	19	5.0	16.23			0.15	28.1
	- 6	0.1	25.4	15	7.6	8.39			0.15	28.1
	- 6	0.1	25.4	11	10.2	3.72			0.15	28.1
	- 6	0.1	25.4	11	12.7	3.68			0.15	28.1
	- 4	0.16	23.5	22	5.0	16.35			0.11	26.6
	- 4	0.16	23.5	20	7.6	7.23			0.11	26.6
	- 4	0.16	23.5	21	10.2	5.45			0.11	26.6
	- 4	0.16	23.5	22	12.7	4.03			0.11	26.6
	- 6	0.15	25.4	22	5.0	13.17			0	31.2
	- 6	0.15	25.4	23	7.6	7.33			0	31.2
	- 6	0.15	25.4	23	10.2	4.98			0	31.2
	- 6	0.15	23	20	12.7	4.17			0	31.2
	- 6	0.15	23	21	12.7	3.38			0	31.2
	- 8	0.14	40	29	7.6	12.18			0.27	23.5
Ft. Greely										
2 Feb 72	- 7	0.2	43	34	11	1.13			2.4	32
3 Feb 72	-12	0.2	51	48	11	1.08			1.6	34
14 Feb 72	-12	0.22	58	48	11	1.05			2.3	36
15 Feb 72	-10	0.22	51	39	11	0.92			1.6	32
Ft. Wainwright										
23 Feb 72	-10	0.2	61	58	11	1.57			2.3	34
24 Feb 72	- 8	0.2	59	56	11	1.31			0.75	38
28 Feb 72	-10	0.21	74	63	11	1.30			2.3	36
1 Mar 72	-10	0.21	56	51	11	1.62			1.3	34
Camp Hale, Colorado										
24 Feb 58 to	- 8	0.25	76	35		0.92				
7 Mar 58	- 7	0.27	77	58		2.26				
	- 6	0.24	74	49		1.99				
	-11	0.27	71	52		2.20				
Wolf Creek Rd. California	- 1	0.36	14	-	24	2.11				
21 March 80	- 1	0.36	14	-	24	1.85				
	- 1	0.36	25.4	12.9	24	1.86				
	- 1	0.38	22	9.3	24	2.28				
Alta, Utah										
26 March 80	0	0.13	6.3	-	24	2.35				
27 March 80	- 3	0.13	24.4	-	24	0.67				
	-.3	0.13	25.9	21	24	1.08				
30 March 80	- 7	0.095	26.7	19.5	24	0.972				
1 April 80	- 1	0.18	30.3	25.5	24	1.42				
**	- 1	0.18	22.9	13.7	20.32	0.720				
**	- 1	0.18	22.9	13.7	20.32	0.682				

* Bennett (1974)

† Large values due to a compacted layer of snow 6 to 10 cm under new snow.

** Plate test

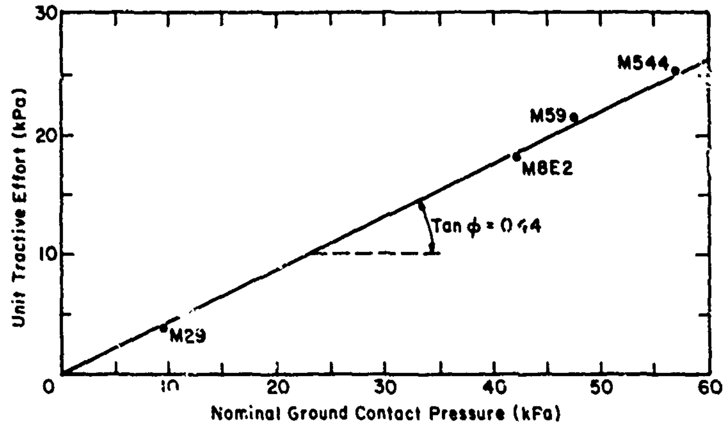


Figure 4. Comparison of nominal ground contact pressure and unit tractive effort.

summary of the Houghton tests) is support of the critical density theory of Gerdel et al. (1954), Diamond and Finelli (1955), and Harrison (1975). The after traffic density measurement of the first pass gave a value of approximately 0.40 for all vehicles regardless of ground pressure. This was true with only two exceptions: one with greater values (the D7 tractor) and the one with lower values (the M29 weasel). Unfortunately, these tests were the only ones from Abele and Parrott (1968) that could be considered as shallow snow tests.

A summary of the results of these tests from the viewpoint of vehicle performance in shallow snow is presented by S.J. Knight as Appendix C in CRREL Technical Report 268 (Harrison 1975). The conclusions were that, while the prediction techniques and equations developed from these data were suitable for predicting performance in deep snow, they were not applicable to the shallow snow problem.

An expedient solution for wheeled vehicles traveling through shallow snow was presented by Harrison (1973) for predicting motion resistance based on snow reaching a critical density of 0.4 Mg m^{-3}. Other pertinent assumptions were that the contact area for a wheeled vehicle would be the projected area under static load for a specific tire pressure. These data are usually available from the tire manufacturer.

The purpose of the approach presented in Harrison (1973) was to compute H_{max} with no consideration given to slip or interface velocity.

To predict gross tractive effort H and motion resistance R, the following equations were suggested.

For gross traction:

$$H = W\mu + Ac' \tag{8}$$

where

W = vehicle weight (or total load on powered wheels, kg)

$\mu = \tan\Phi$ where $0 < \Phi < \phi$ (angle of interface friction)

A = nominal contact area (static tire print for wheels, m^2)

$c' = c \dfrac{\tan\Phi}{\tan\phi}$ (adhesion) $0 < c' < c$ (kPa^2)

For sinkage:

$$z = (1 - \frac{\gamma}{0.4})\, h \tag{9}$$

where

h = snow depth (m)

γ = equivalent water content of snow (density in Mg m^{-3}).

For compaction resistance:

$$R_c = \frac{bkz^{n+1}}{n+1} \tag{10}$$

$R_c = 0.38\, bp^{0.625}\, z^{1.975}$ at $n = 1.6$

b = track or tire width

k = p/z^n

p = contact pressure

For bulldozing resistance:

$$R_b = (RN_w + \gamma z^2 N_{\gamma w} + czN_{caw})\, 2b \tag{11}$$

$R = \gamma z^2 N_\gamma + czN_{ca}$ (N/m)

γ = snow density (Mg m^{-3})

z = sinkage (m)

c = snow cohesion (kPa)

$$\text{"}N\text{" factors} = f(\phi; \rho) \qquad (12)$$

where ϕ = snow internal friction angle
ρ = (90° + approach angle of track) or [90° + arc cos $(z/\sqrt{zd})$ for wheeled vehicles]
d = tire diameter (m)

Equation 10 for compaction resistance is a modification of the deep snow tracked vehicle equation used by the Land Locomotion Laboratory (Harrison and Czako 1961). It was based on a selected value of n = 1.6 which was representative of a number of experiments in deep snow.

Equation 11 for bulldozing resistance was adapted from the work of Reece (1965) and Hettiaratchi (1968). The curves for determining the "N" factors are contained in Hettiaratchi's work but are not considered relevant for inclusion or further discussion.

Liston (1974a, 1974b) presented an interesting approach that uses the sinkage of a plate which loads the snow cover to the same pressure as would a tire having a specified diameter, width and inflation pressure. An abridged version of his approach follows.

It is assumed that the maximum shear strength of snow can be described by the Mohr-Coulomb equation:

$$\tau = c + p \tan\phi \qquad (13)$$

where τ = shear stress (psi)
τ = shear stress (psi)
c = apparent cohesion (psi)
p = normal pressure (psi)
ϕ = angle of internal friction.

In determining the magnitude of c and ϕ, a device having a rubber surface (rather than a roughened surface) is used to impose a failure surface within the snow mass. The more sophisticated equation of Hanamoto and Janosi (1961), which states that

$$\tau = (c + p \tan\phi)(1 - e^{-j/k}), \qquad (14)$$

was not used because it was found that the magnitude of k was small, on the order of 0.02 m, so that maximum shear strength was developed along most of the tire/snow interface.

The contact area was assumed to consist of two sections: 1) an area A_p beginning at the uppermost contact point between the wheel and snow and ending at the bottom of the rut (see Figs. 5 and 6), and 2) an area A consisting of the flat contact patch at the bottom of the tire. A_p is identified as the peripheral area and A as the contact area.

In order to compute the peripheral area A_p, it is noted that

$$dA_p = br\, d\theta \qquad (15)$$

as shown in Figure 5, and

$$A_p = br \int_0^{\theta_0} d\theta = br\, \theta_0. \qquad (16)$$

Also

$$\theta_0 = \cos^{-1}\left(\frac{r - z_0}{r}\right) \qquad (17)$$

giving

$$A = br \cos^{-1}\left(\frac{r - z_0}{r}\right) \qquad (18)$$

In computing the contact area (A), it is assumed that

$$W = p_l A \qquad (19)$$

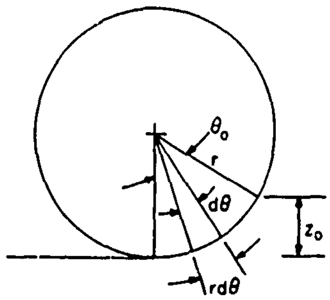


Figure 5. Determination of wheel/snow interface area (from Liston 1974a, 1974b).

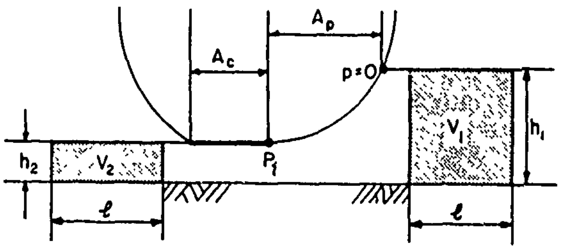


Figure 6. Volume change produced by passage of wheel and identification of contact areas.

where

p_i = inflation pressure (psi)
A = contact area (in.^{2})

Equation 19 is inaccurate as it does not account for the support provided by the tire carcass. The total contact area A_t per wheel is taken as

$$A_t = (A_p + A) \tag{20}$$

and gross traction is found from

$$H = (A_p + A)\, c + W \tan\phi . \tag{21}$$

In determining motion resistance, it was assumed that the work expended in compacting the snow was the single source of resistance offered by the snow. The following additional assumptions were made:

1. As shown in Figure 6, the only dimensional change that occurs when the snow is compacted from its original volume V_1 to its final volume V_2 is in the snow depth h.

2. The pressure along the interface of the wheel and snow is zero at the point of contact and is equal to the inflation pressure at the lowest point of contact.

3. The variation in pressure along the wheel/snow interface is hyperbolic in form. The pressure distribution is depicted in Figure 7.

The work expended in compacting the snow from V_1 to V_2 is

$$E = \int_{V_1}^{V_2} p\, dv. \tag{22}$$

The assumption that the pressure p varies hyperbolically leads to the following relationship:

$$p = p_i \frac{V_2}{V} \tag{23}$$

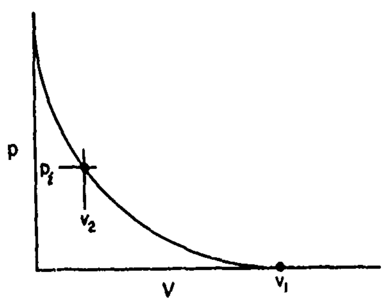


Figure 7. Assumed pressure distribution along wheel/snow interface.

in which

p_i = the inflation pressure
V_1 = $bh_1\ell$ = the initial volume of a snow mass
V_2 = $bh_2\ell$ = the final volume of the same snow mass

Introducing eq 24 into eq 23 and completing the integration gives the following expression for compaction energy E:

$$E = p_i V_2 \ln \frac{V_2}{V_1} .$$

If R_c is defined as the force resisting motion which is related to compaction, then work can be expressed by multiplying R_c by ℓ. Also, the volume can be expressed as $bh_1\ell$. Substituting the above expressions for E and V gives the equation

$$R_c = p_i b h_2 \ln \frac{h_2}{h_1} . \tag{25}$$

The total motion resistance R_t is obtained by adding the compaction resistance to the hard surface rolling resistance R_H. In order to predict net tractive effort H_n, which is the parameter of interest, the motion resistance is subtracted from the gross traction:

$$H_n = H - R_H - 2\,(R_{c1} + R_{c2}) \tag{26}$$

in which R_{c1} and R_{c2} are the resistances of the front and rear wheels, respectively. Values of h_2 are obtained from plate tests where the contact pressure of the plate and the tire are approximately equal.

Harrison (1975) presented a method for determining the motion resistance of vehicles in shallow snow based on the determination of a unit energy constant. The gross tractive effort is found by the use of a now-familiar expression:

$$H = Ac_a + W \tan\phi \tag{27}$$

where A is the total contact area of the traction elements and W is the vehicle weight.

Motion resistance over shallow snow is defined as the work performed by the vehicle in compacting undisturbed snow when forming wheel ruts. Figure 8 shows the stress-deformation curve of a column of snow under simple compression.

The equation for work E is stated simply as

$$E = A \int_0^{z_e} \sigma\, dz \tag{28}$$

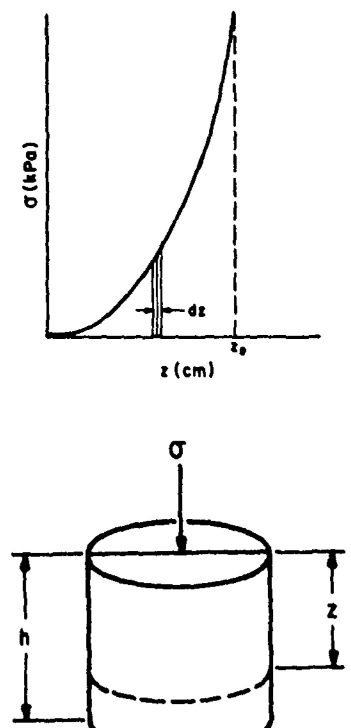


Figure 8. Column of snow under simple compression.

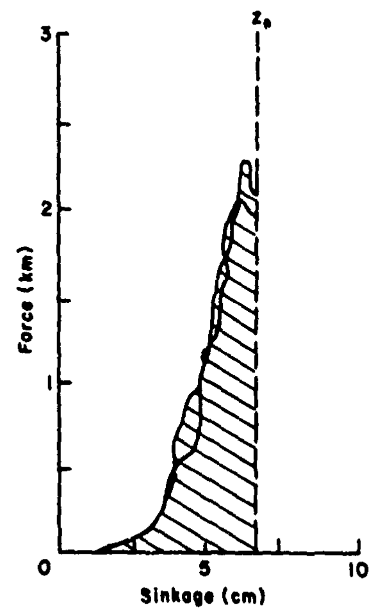


Figure 9. Plate sinkage tests (Harrison 1975).

which is basically the same as those of Liston (1974), Bekker (1956), Nuttall (1957), etc. The characteristics of the curve differ from simple compression in a plastic medium in that there is compaction throughout deformation z rather than plastic flow as described by the typical soil model. In addition, the deformation z approaches a maximum value z_e as maximum compaction (critical density) is approached and the $\sigma - z$ curve becomes asymptotic. Critical density is assumed to be approximately 0.5 Mg m^{-3}. This value of γ' is considered acceptable for two reasons: first, the additional deformation between $\gamma = 0.5$ and $\gamma = 1.0$ is minute and the increase in σ required is quite large, and secondly, in the unconfined state that exists, plastic flow will occur at this density rather than a decrease in volume (Kinoshita and Akitaya 1970).

The curves in Figure 9 illustrate the use of z_e. The effective sinkage (z_e) is dependent on the depth at which ½ γ_{max} occurs and is easily computed from the following equation for predicting sinkage (Harrison 1973):

$$z_e = (1 - \frac{\gamma}{\gamma'})h \qquad (29)$$

where γ' is the selected value of post-compaction density. In practice, the measured curves are finalized (when necessary) by locating the z_e line and constructing the finished curve so that it becomes asymptotic with z_e (Fig. 9).

The area beneath the load-sinkage curve in Figure 9 represents the total work E expended in compacting the snow from $z = 0$ to $z = z_e$. To obtain the work of compaction per unit volume ω the total work E is divided by the snow depth h and the area of the circular plate A_p used to obtain the curves:

$$\omega = \frac{E}{h\,A_p} \qquad (30)$$

To minimize the effect of size, a minimum plate diameter of 20 cm is used.

The resistance to motion from snow compaction is given by the equation

$$R_c = 2b\omega h \qquad (31)$$

where b is the effective wheel width (or track width).

Bekker (1976) presented a method of determining rolling resistance based upon the shallow snow tests conducted by Harrison (1975). The method of solution combined the earlier efforts of Bekker and Semonin (1975) for determining the rolling resistance due to tire flexing and those of Bekker (1973) for determining the resistance to motion by vehicles traveling in a layered medium.

To determine the snow resistance to motion, Bekker utilizes a multi-layered approach based on the characteristics of plate sinkage curves in shallow

snow. As shown in Figure 10, the $p - z$ curve is plotted in log-log form and the "zones" are selected dependent on obvious changes in slope "n." Each zone produces a set of k_c, k_ϕ, and n values by which the resistance for that zone can be calculated.

For "zone 1" where n is generally less than 3.0 or for any zone where $n < 3.0$, the resistance is determined by

$$R_c = \frac{b_0 k z_1^{(n+1)}}{n+1} \qquad (32)$$

where z is determined from the expression:

$$z_1 = \left(\frac{3W_1}{bk\sqrt{D} \quad (3-n)} \right)^{\left(\frac{2}{2n+1}\right)} \qquad (33)$$

From Bekker (1976)

$$W_1 = 0.3\, b_0 k \quad (3-n) \quad \sqrt{D} \quad z_1^{\left(\frac{2n+1}{2}\right)} \qquad (34)$$

Whenever there is a zone where $n > 3.0$, the "full solution of the integral $W = b_0 \int_{z_0}^{z} p_x\, dx$ must be used" (Bekker 1976) so that

$$W_j = b_0 k_j \quad \sqrt{D} \left[z_j \, \frac{2n_j + 1}{2} \left(a_j - b_j \quad \frac{2z_j}{D} \right) \right.$$

$$\left. - z_{j-1} \, \frac{2n_j + 1}{2} \left(a_j - b_j \quad \frac{hz_j - 1}{D} \right) \right]$$

where

$$a_j = (1 - 0.509\, n_j + 0.222\, n_j^2 - 0.052\, n_j^3 + + 0{,}005\, n_j^4) \qquad (36)$$

and

$$b_j = (0.25 - 0.26\, n_j + 0.137\, n_j^2 - 0.028\, n_j^3) \qquad (37)$$

R_c is then the sum of R_I, R_{II}... R_j and the total resistance for a wheeled vehicle is

$$R = 2\,(R_c + R_t) + n_w R_t \qquad (38)$$

where n_w is the number of trailing wheels, i.e. 2 for a 4 x 4, 6 for an 8 x 8, etc.

Nuttall et al. (1975) developed an ad-hoc shallow snow model for wheels and tracks. The model is used as part of the Army Mobility Model to predict traction and resistance of a snow cover deposited on frozen soil.

The model computes maximum traction H and resistance R. Net traction is determined as the difference between H and R:

$$H = cA + W\mu \qquad (39)$$

where

c = interface shearing resistance (psi)
A = contact area (in.^{2})
W = vehicle weight (lb)
μ = tangent of the angle of interface shearing resistance.

For wheels the following expression is used:

$$\frac{R}{W} = \frac{10}{n_a} \quad \frac{n_t b}{d} \quad \frac{\gamma h}{\ell} \qquad (40)$$

where

$\ell = 2\sqrt{\delta d - \delta^2}$ (in.)
d = tire diameter (undeflected) (in.)

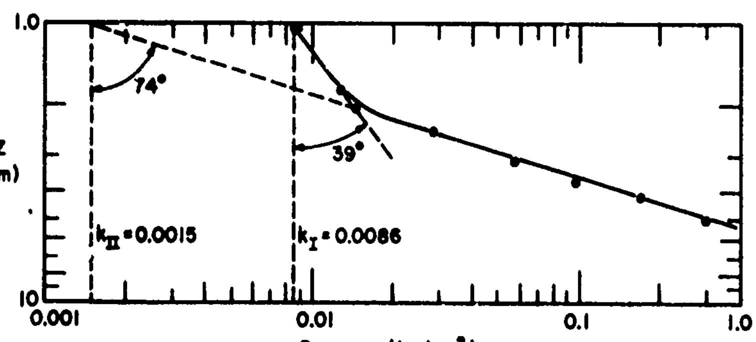


Figure 10. Pressure-sinkage graph of load-sinkage function in shallow snow. Snow depth is approximately 9 cm, temperature is - 1°C, and plate diameter about 20.3 cm (adapted from Harrison's 1975 data).

δ = tire deflection (in.)
n_t = maximum number of tires per axle
n_a = number of axles
b = tire section width (in.)
h = snow depth (in.)
γ = snow specific weight

For tracks, Nuttall (1957) uses a simpler expression

$$\frac{R}{W} = \left(\frac{h}{L} - 0.15\right) \tag{41}$$

where L = overall track length (in.)

The relationships were developed by Nuttall from a compilation of experimental test data available in the literature.

Table 2 gives a comparison of predictions by four models. These predictions were made from past data relative to vehicle tests in shallow snow as presented in Liston (1974a, 1974b), Harrison (1975) and Nuttall (1975). The prediction by the Bekker model was furnished the author by private communication with Dr. Bekker.

The high value produced by the Liston model for the Hanover tests results from the model's characteristic of inflation pressure dependency.

It can easily be shown that the Liston (1974), Bekker (1976), and Nuttall (1957) models (since all are compaction energy based) can be resolved into eq 31, $R_c = 2b\omega h$, which is the Harrison (1975) model with the parameter ω, the work of compaction per unit volume, as the basic transfer mechanism.

MODEL SELECTION

If the model to be proposed in this report is to be selected on review of the past work, which must be considered as the state-of-the-art, then a number of guidelines or goals should be contemplated.

Before facing this selection process, let us review some common aspects of the past work. All methods discussed in the review proposed the use of some form of the Mo' -Coulomb relationship for predicting the tractive force developed by a vehicle. Further, most methods proposed that the energy dissipated in making the rut in snow be considered as the resistance to motion caused by compacting the snow. The most apparent weakness of all the methods is lack of validation by field tests.

The guidelines for model selection under the above-stated circumstances should therefore be based on the following:

1. The equations for traction and snow resistance to motion should clearly address the problem. These equations should be sufficiently fundamental that future modification is possible.
2. The parameters which reflect snow strength characteristics should facilitate prediction from basic snowpack properties and be obtainable by field measurement without great difficulty.
3. Because of the lack of a data bank on shallow snow-vehicle performance parameters, any proposed model must be considered provisional at best.

Traction

The prediction of gross tractive effort is based on the Mohr-Coulomb expression:

$$\tau = \sigma \tan\phi + c$$

where τ is shearing stress (kPa), σ is normal stress (kPa),

Table 2. Comparison of prediction models.

Vehicle	Location	Computed values of R (N)*				Measured values (N)		
		Liston	Harrison 1975	Nuttall 1975	Bekker	R	R_H	R_c
M151A1	Silver Creek Bridge (30 Jan 73)	2594	2280	1584		2520†	373†	2146
Chevrolet Carryall	Hanover, N. H. (18 Feb 75)	1628	636	609	540	1237†	657†	580
M151A1	Houghton, Mich. (18 Feb 76)	898	884	1365		1490**	636**	853
M151A1	Houghton, Mich. (30 Jan 75)	862	643	676		1365**	636**	729

* Compaction resistance $R_c = R - R_I$
† Measured by deceleration method
** Measured by towing

and c and ϕ are internal shearing resistance and the associated angle of shearing resistance, respectively. As used in most vehicle performance models, this equation is written in the form for predicting gross tractive force H:

$$H = Ac + W \tan\phi \quad (43)$$

for vehicles with chains or grousers. For vehicles with pneumatic tires or band tracks with no grousers,

$$H = A\, c_a + W \tan\phi \quad (44)$$

where A and W are the contact area and vehicle weight, and c_a and θ are the interface shearing resistance and associated angle of interface shearing resistance, respectively. The prediction of gross traction H will be in terms of maximum sustained force rather than a traction-slip function.

To accommodate traction aids in the model, increased traction will be reflected by multiplying the results of a standard tire by coefficients obtained from experimental tests (Table 3) of Liston (1977).

The determination of total traction developed will be a summation of the traction developed by each traction element.

Resistance

The energetics relationship (Harrison 1975) which states

$$R_c = 2\, b\omega h \quad (31)$$

is proposed as the resistance model. The relationship between ω, snow temperature T, and snow density γ shown in Figure 11 requires further validating by a more complete data bank than shown in Tables 1 and 4, on which Figure 11 was based.

Slush and thawing soils

An ad-hoc model is suggested for slush and thawing soils until a data bank has been established. These two materials, due to their characteristics of flowing rather than compacting when disturbed, will be treated as surcharge models having hydrostatic properties relative to the forces required to cause flow. It is assumed that the traction is obtained from the firm supporting layer. The base properties for slush will be packed snow at -1°C (ice), and a layer having a high degree of unfrozen water for thawing soil. The resistance force is determined as follows:

$$R_c = \frac{\gamma h^2}{2} \quad (45)$$

where γ is the density of the layer and h is the layer thickness. Values of γ are as follows:

slush: $\gamma = 0.75$ g/cm³ (750 kg/m³)
thawing soil layers: $\gamma = 1760$ to 2100 kg/m³

with loams on the lower end and sands at the higher values. This relationship is loosely based on the unlikely premise that there will be no sinkage or breakthrough in the supporting layer.

Ice, hard-packed snow, packed snow (new)

These surface conditions are assumed to have negligible motion resistance. The emphasis in predicting performance is placed on the following equations:

$$H = W \tan\phi + Ac \quad (43)$$

or

$$H = W \tan\Phi + Ac_a \quad (44)$$

where μ ($\tan\phi$ or $\tan\Phi$) in this context is determined from experimental data. Figures 12 and 13 show values of μ tabulated from a large number of experiments, as functions of bearing capacity and snow surface temperature. Figure 14 gives values of μ for ice relative to surface temperature. A summary of Figures 12, 13, and 14 is given in Table 5.

Table 3. Traction aid data

Tire	Pressure (kPa)	Drawbar pull (N)	Percent increase over NDCC standard	Traction aid coefficient
NDCC standard	55	2420	0	1
NDCC standard with chains	103	3372	60	1.60
NDCC radial	55	2535	25	1.25
Bias-ply snow	55	3056	35	1.35
Bias-ply snow with studs	55	3252	45	1.45
Summer radial	55	3016	45	1.45
Sand tire	55	2304	25	1.25

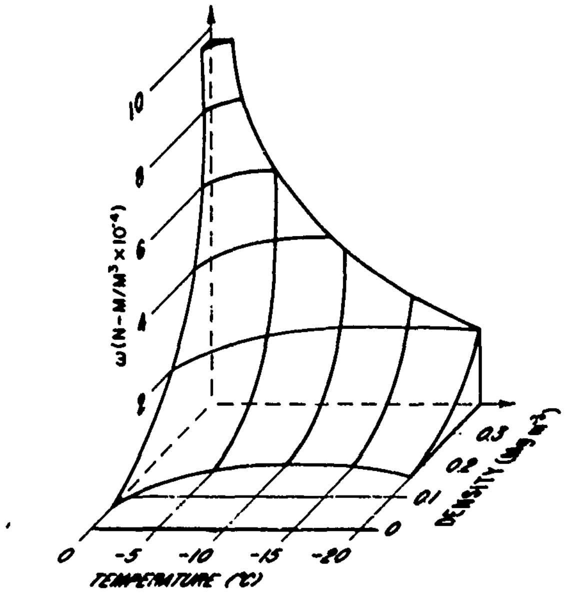


Figure 11. ω versus temperature and density.

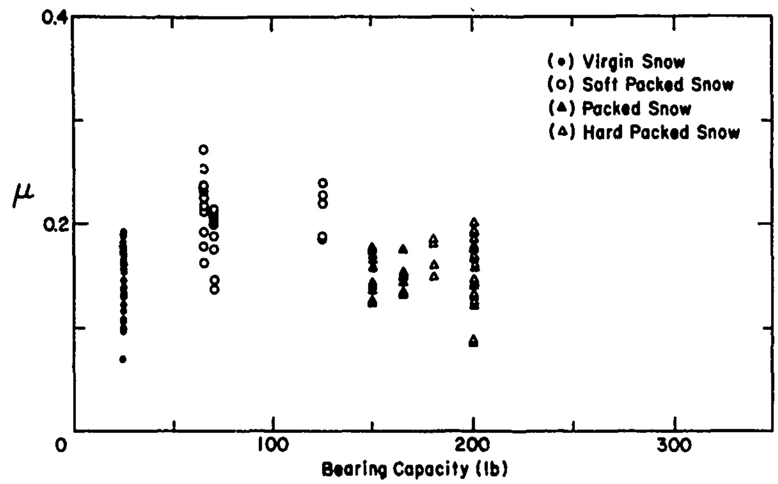


Figure 12. μ versus bearing capacity—disturbed snow.

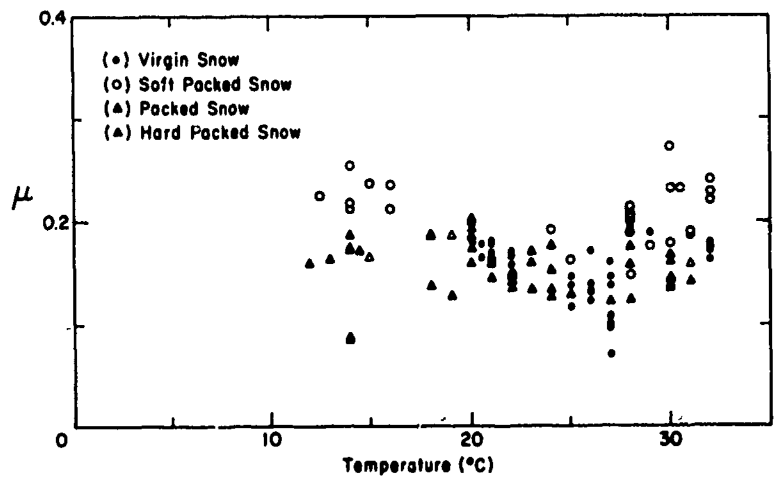


Figure 13. μ versus temperature—disturbed snow.

Table 4. Shallow snow data – vehicle characteristics.

Location	Vehicle type	h (cm)	d (cm)	b (cm)	W (Kg)	P kPa (psi)	D (N)	R (N)
Houghton								
30 Jan 75	M151	15	75	20	1,410	137 (20)	1,560	1,312
		13	75	21	1,410	103 (15)	1,640	1,267
		15	75	20	1,410	103 (15)	1,470	1,036
		13	75	22	1,410	55 (8)	2,090	1,365
		14	75	21	1,410	55 (8)	1,870	1,352
19 Feb 75	M151	8	75	22	1,410	103 (15)	1,380	720
		16	75		1,410	103 (15)		1,468
		8	75	23	1,410	55 (8)	1,650	854
3 Mar 75	M151	12	75	21	1,410	103 (15)	2,490	1,316
		12.7	75	22	1,410	55 (8)	2,220	1,556
12 Feb 75	M151	14	75	22	1,410	103 (15)	2,000	1,321
		15	75	24	1,410	55 (8)	2,220	1,374
4 Feb 76	ENGESA	10	775[1]			172 (25)		3,430
5 Feb 76	2½-ton	14	775[1]			172 (25)		4,400
18 Feb 76	M151A1	18	353[1]		1,560	55 (8)		1,490
		14	353[1]		1,560	55 (8)		1,490
19 Feb 76	ENGESA 2½-ton	16	775[1]			172 (25)		3,700
Ft. Greely								
2 Feb 72	M113A1	43	280[2]	38	9,500	43.8 (6.3)	34,000	
3 Feb 72		51	280[2]	38	9,500	43.8 (6.3)	39,000	
14 Feb 72		58	280[2]	38	9,500	43.8 (6.3)	38,000	
15 Feb 72		51	280[2]	38	9,500	43.8 (6.3)	40,000	
23 Feb 72		61	280[2]	38	9,500	43.8 (6.3)	32,000	
24 Feb 72		58	280[2]	38	9,500	43.8 (6.3)	34,000	
28 Feb 72		74	280[2]	38	9,500	43.8 (6.3)	29,000	
1 Mar 72		56	280[2]	38	9,500	43.8 (6.3)	31,000	
Silver Creek Bridge								
30 Jan 73	M151A1	24	75	19	1,452	55 (8)	2,320	1,214
		24	75	19	1,452	103 (20)	1,630	2,520
27 Jan 73	M151A1	24	75	19	1,452	103 (20)	2,380	1,970
31 Jan 73	M34		107	29	7,720	241 (35)	3,400	790
Camp Hale, Colorado								
24 Feb 58 to	M29	76	198[2]	51	1,950	9.6 (1.4)	5,100	5,700[3]
7 Mar 58	M5A4	77	300[2]	31	9,350	58 (8.4)	22,000	31,808
	M59	74	235[2]	53	17,600	48 (7.0)	56,000	29,100
	M8EZ	71	406[2]	53	18,900	41 (6.0)	58,000	35,500

[1] Actual tire contact area (cm^2)
[2] Track length (cm).
[3] Vehicle dragging.

Table 5. Mechanical properties for ice and packed snow.

Parameter	Ice	Packed snow
$\tan\phi'$	0.07	0.23
$\tan\Phi$	0.03	0.14
c (kPa)	0	0.3
c_a (kPa)	0	0.6

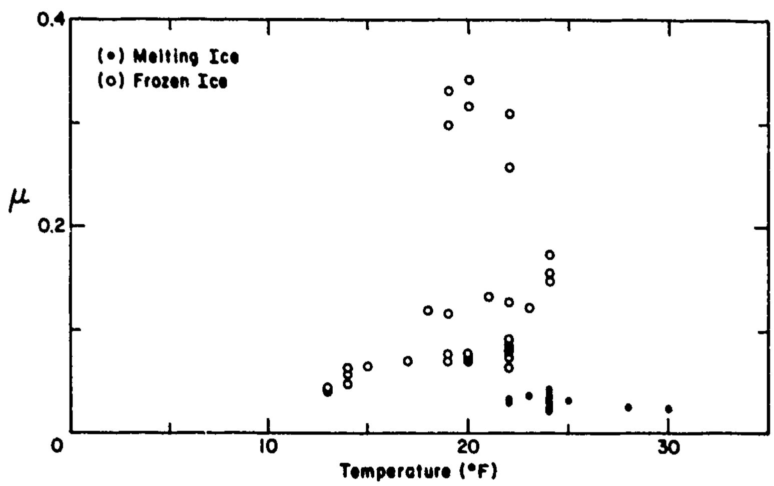


Figure 14. μ versus temperature – ice.

Within the temperature ranges of the experiments, the data scatter was significant with no definite μ vs temperature relationships apparent. If data were available for temperature ranges well below 0°F, perhaps some trends would have been evident.

The contact area A for pneumatic tires will be determined by the tire inflation pressure as follows:

$$A = \frac{W}{P_g} \tag{45}$$

where W is the wheel load and P_g is the tire inflation pressure, and by the width-length product for tracked vehicles:

$$A = 2b\ell \tag{46}$$

where b and ℓ are the nominal width and length measurements.

Recommendation for use of eq 45 is based on Table 6 which shows values of measured tire contact areas divided by wheel load vs predictions using this equation. The average error is 10% for eq 45 over a number of inflation pressures and tire structural characteristics. A 10% average error is quite acceptable at this time.

River and lake ice

The safe crossing thickness of river or lake ice, i.e. freshwater ice, can be determined from the following equation (Johnson 1979):

$$h = C_c \, \sigma_t^{-0.6} \tag{47}$$

where h = ice thickness, C_c = vehicle characteristic constant and σ_t = tensile strength. A reasonable tensile strength for ice during winter conditions is 980 kPa (Nevel 1978), and in springtime this value deteriorates to 100 kPa.

Johnson (1979) gives the following values of C_c for the M151, M35A2, M113 and M60A1 Army vehicles. All values are for highway or combat loads: M151–8.1 kPa, M35A2–22.4 kPa, M113–23.0 kPa, M60A1–51.7 kPa. Thus, for example, during winter conditions, the safe thickness of ice for the vehicles as listed will be: M155–0.13 m, M35A2–0.36 m, M113–0.37m, M60A1–0.83 m.

Model use

The shallow snow model consists of the relationships given eq 42-47. Where possible, values of surface strength parameters have been, or will be, furnished from experimental tests. When this is not possible, the values will be predicted. Toward this end, a preliminary study was made (Berger in prep.). These studies will be later expanded in scope to include the forecasting of all parameters including packed snow, slush, thawing soils and ice.

Values of H and R are required by the Army Mobility Model (Nuttall et al. 1975). The decision-making process for obtaining these values by using eq 42-46 (and the associated tables) is in fact the shallow surface layer portion of the eventual "Cold Regions Mobility Model." This is intended for use as a submodel to the Army Mobility Model and associated models. The following flow diagrams and examples present an algorithm for the decision-making processes for determining H and R.

Factors to be considered in the algorithm are:

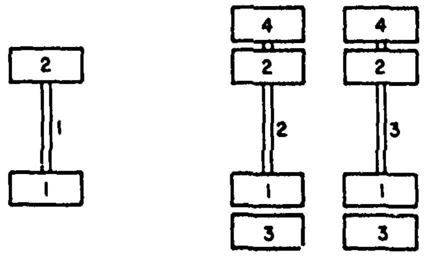
1. Number and arrangement of traction elements (i.e. driven, towed, duals).
2. Characteristics of traction elements: (i.e. smooth "rubber," grousered, inflation pressure, chains, studs, ply design) (see Table3).
3. Surface material: (see Tables 5 and 7).

Table 6. Tire contact area (in.^{2}).

Measured	*Theoretical*	Δ	*% Error*
52.6	53.9	1.3	2.5
45.2	49.3	4.1	9.1
46.1	49.3	3.2	6.9
50.3	53.9	3.6	7.2
49.5	49.3	+0.2	0.4
52.6	53.9	1.3	2.5
45.0	49.3	4.3	9.6
49.5	53.9	4.4	8.9
44.0	49.3	5.3	12.0
48.5	53.9	5.4	11.1
50.3	53.9	3.6	7.2
50.9	53.9	3.0	5.9
54.2	53.9	+0.3	0.6
49.2	53.9	4.7	9.6
49.3	53.9	4.6	9.3
46.2	53.9	7.7	16.7
41.5	49.3	7.8	18.8
48.1	53.9	5.8	12.1
47.0	53.9	6.9	14.7
45.7	53.9	8.2	17.9
46.8	53.9	7.1	15.2
45.1	53.9	8.8	19.5
48.0	53.9	5.9	12.3
		Avg.	10.0%

i = axle set number
j = tire number
Number of units (axle sets) = N
Driven axle = DR(i,j) (0 - not driven, 1 - driven)
Weight per wheel = W(i,j)
Tire diameter = D(i,j)
Tire width = B(i,j)
Tire pressure = P(i,j)

Example: 6 x 6 with duals



Weight Matrix:

$$\begin{matrix} W_{11} & W_{12} & 0 & 0 \\ W_{21} & W_{22} & W_{23} & W_{24} \\ W_{31} & W_{32} & W_{33} & W_{34} \end{matrix}$$

Figure 15. Inputs for wheeled vehicles.

Actual programming of the model has not yet been undertaken. A description of the model can be divided into two portions, the input data structure and the various subprograms which calculate traction and motion resistance.

An explanation of the data structure involves a description of all the pertinent variables and how they are input into the model. The computer model breaks the vehicle down into a series of units. Each transverse set of axles or set of tracks is considered a unit. The variables pertaining to the geometry of wheeled and tracked vehicles are shown in Figures 15 and 16 respectively. Data for particular tire or track characteristics are indexed into a two dimensional array. Each of the vehicle units is represented by a row in the array and the columns indicate location on the unit.

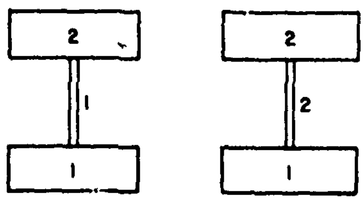
For calculating traction, separate models have been developed for tracked and wheeled vehicles. The traction subprogram (Fig. 17 and 18) simply sums the traction values for each driven unit on the vehicle.

Resistance is calculated by the motion resistance subprogram (Fig. 19). It is based on eq 31 and uses ω values.

No driver program has been assembled to coordinate the various subprograms. Basically, the driver program should merely establish a user interface for data input and output and call each of the subprograms to perform its function.

i = track set number
j = track number
Number of units (track sets) = N
Driven axle = DR(i,j) (0- not driven, 1 - driven)
Track length = L(i,j)
Track width = (B(i,j)
Weight per track = W(i,j)

Example: BV-202



Weight Matrix:

$$\begin{matrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{matrix}$$

Figure 16. Inputs for tracked vehicles.

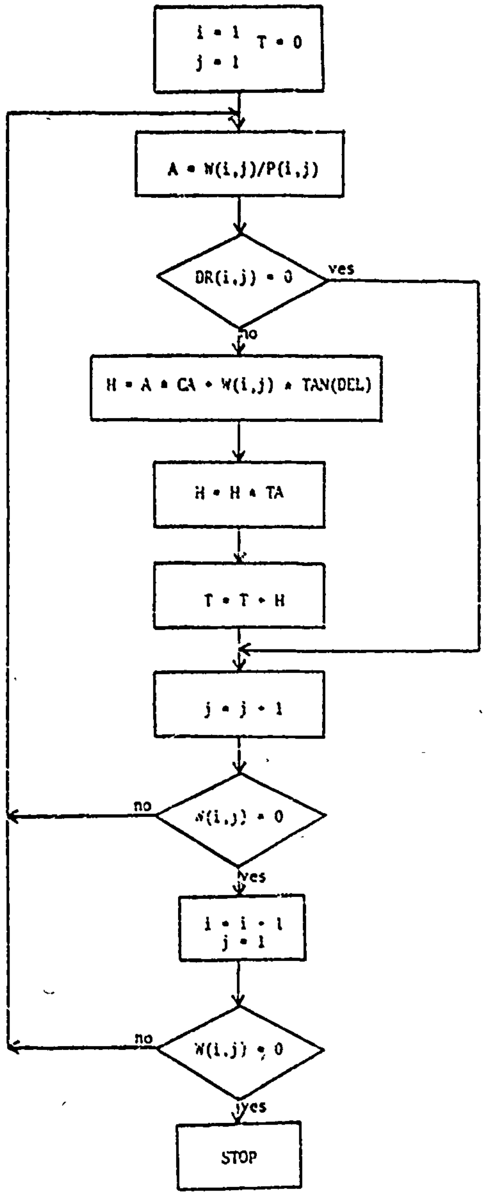


Figure 17. Traction subprogram for wheeled vehicles.

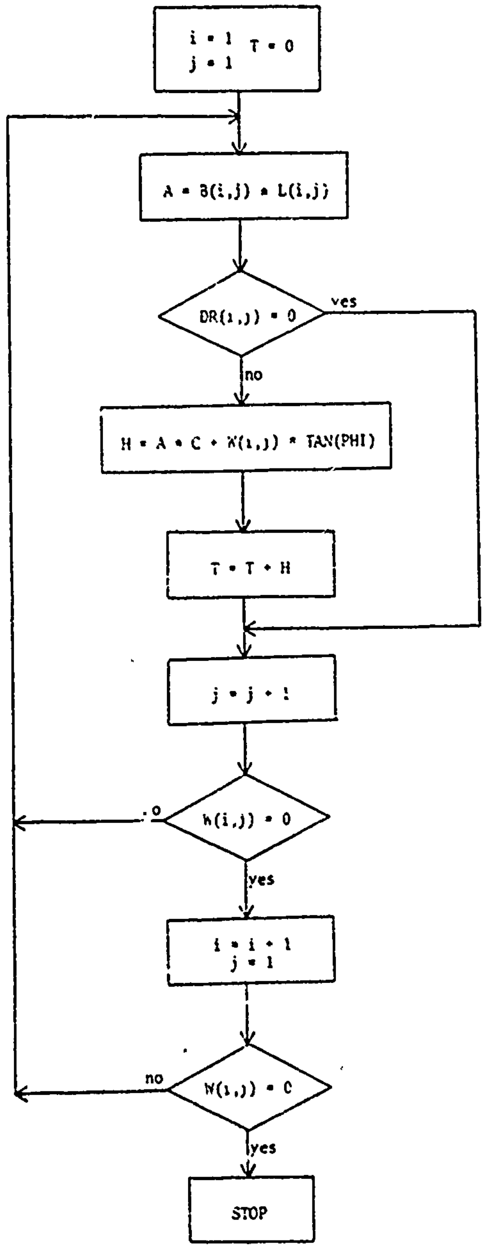


Figure 18. Traction subprogram for tracked vehicles.

Table 7. Mechanical properties of shallow snow.

Snow type	Snow temp (°C)	Air temp (°C)	ρ (g/cm³)	ca (kPa)	c (kPa)	δ (degrees)	φ (degrees)	ω (J/cm³)	ρ (critical) (g/cm³)
New Snow									
Dry[1]	-5 to -10	-9	0.15	0	1 to 1.5	18 to 20	18 to 20	0.4 to 0.6	
Dry[4]	-10 to -20	-15	0.2 to 0.26	0	0.5	22 to 24	26	0.7 to 0.8	
Wet[5]	-1 to -3	-1		1 to 1.2	—	25 to 30	—	1 to 1.2	
Dry[a]	-3	-6	0.13					0.67 to 1.08	
Dry[b]	-7	-10	0.095					0.972	
Settling snow									
Powder[1]	-3 to -5	-8	0.1 to 0.2	0	3.9	20	22	0.75	
Bonded[2]	-3 to -5	-7	0.3	0	—	20	—	0.6	
Bonded[3]	-3 to -5	-15	0.23		6.8	25	25	0.7	
Powder[c]	0	-2	0.13					2.35	
Powder[d]	-1	0	0.18					0.68 to 0.72	
Powder[e]	-1	0	0.18					1.4[illegible]	
Old snow									
Crusted[6]	-10	-13	0.21		0		[illegible]		
Crusted[6]	-5	-10	0.2		0.68		20		
Compacted[6]	-4 to -10	-1	0.4		3.0		20	~6	
Compacted[6]	-4 to -10	+6	0.4		0		30	~4	
Corn snow	0	>0	0.6		0				
Slush	0	>0	0.7		0				
Crusted[f]	-1	+3	0.36					1.85 to 2.11	
Crusted[g]	-1	+3	0.38					2.28	

[1]Houghton, 30 Jan 75.
[2]Houghton, 3 Mar 75.
[3]Alaska, Ft. Greely and Ft. Wainwright, Feb 72.
[4]Houghton, 4 Feb 76, 5 Feb 76.
[5]Houghton, 18 Feb 76.
[6]Silver Creek Bridge, Feb 73.
[a]Alta, Utah, 27 March 80 (vehicle test)
[b]Alta, 30 March 80 (vehicle test)
[c]Alta, 26 March 80 (vehicle test)
[d]Alta, 1 April 80
[e]Alta, 1 April 80 (vehicle test)
[f]Wolf Creek, California, 21 March 80 (vehicle test)
[g]Wolf Creek, 21 March 80 (vehicle test)

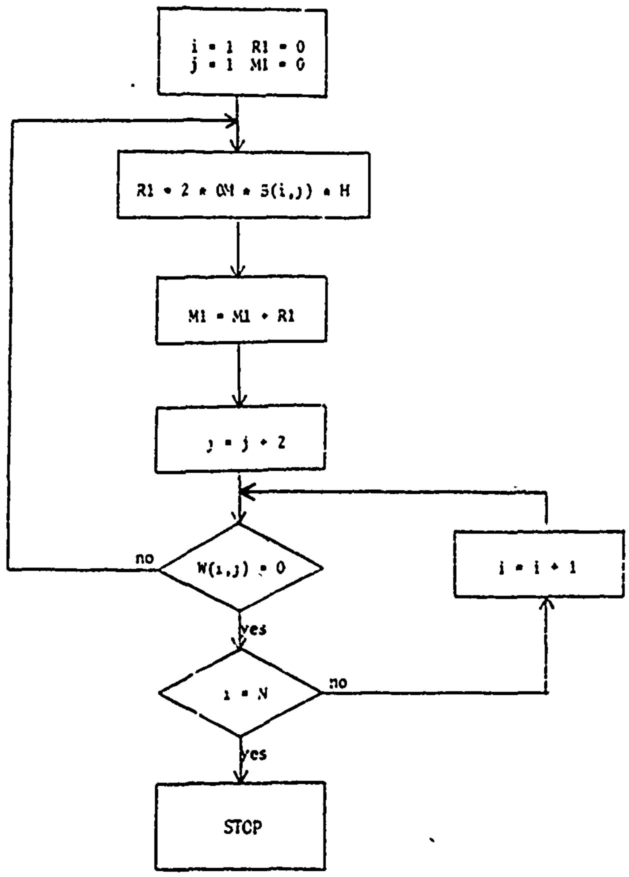


Figure 19. Motion resistance subprogram.

CONCLUSIONS

Most tests and measurements reported were conducted under conditions as they occurred during a scheduled test period. This led in most cases to results that were nongeneric to the terminology used; i.e. shallow snow, packed snow, etc. In most cases, the shallow snow layers were complex in nature and varied considerably from site to site. The scatter in ice and packed snow data is also indicative of variations not accounted for.

It is therefore a reasonable conclusion that there is only a small amount of information available relative to shallow surface layers in cold regions and vehicle performance.

Projects are now in progress to establish a comprehensive data bank over the next three years to correct this deficiency. Additional studies are planned to establish more accurate vehicle performance models for predicting mobility in winter environments.

SELECTED BIBLIOGRAPHY

Abele, G. (1970) Deformation of snow under rigid plates at a constant rate of penetration. U.S.A. Cold Regions Research and Engineering Laboratory (CRREL) Research Report 273. AD704708.

Abele, G. and W.H. Parrott (1968) A study of track grouser penetration into hard snow and snow adhesions to vehicle tracks. CRREL Interim Report (unpublished).

Abele, G. and J. Dufendach (1970) Arctic trafficability research studies (partial report). U.S. Army Arctic Test Center, Seattle, Washington.

Abele, G. and D. Mouret (1971) Arctic trafficability research studies (partial report). U.S. Army Arctic Test Center, Seattle, Washington.

Bader, H. and D. Kuroiwa (1962) Physics and mechanics of snow as a material. CRREL Cold Regions Science and Engineering Monograph II-B. AD287052.

Bekker, M.G. (1956) *Theory of land locomotion.* Ann Arbor: University of Michigan Press.

Bekker, M.G. (1973) Tire selection for off-road vehicles. *Design News,* December.

Bekker, M.G. (1976) Rolling resistance of pneumatic tires in shallow snow covering hard surfaces (unpublished). Memorandum submitted to CRREL.

Bekker, M.G. and E. Semonin (1975) Motion resistance of pneumatic tires. *Journal of Automotive Engineering,* no. 4.

Bennett, W.D. (1973) Documentation of snow characteristics for over snow vehicle validation tests. CRREL Technical Note (unpublished).

Bennett, W.D. (1974) Results of snow measurements from mobility test program. CRREL Technical Note (unpublished).

Bennett, W.D. (1975) Description of a snow strength measuring device. CRREL Internal Report 443 (unpublished).

Berger, R.H. (in prep.) A feasibility study for developing a snowpack model for use in predicting mobility parameters. CRREL Report.

Blackmon, C.A. and A.A. Rula (1960) Tests on subarctic snow. USAEWES Technical Memo 3-414, Report 4.

Bogorodskii, V.V. (Ed.) (1976) Physical methods of studying ice and snow. CRREL Draft Translation 539. ADA 630818.

Diamond, M., H. Bader and J.L. Langon (1959) Studies on vehicular trafficability of snow, Parts 1 and 2. Snow, Ice and Permafrost Research Establishment Technical Report 35. AD099687.

Gerdel, R.W., W.H. Parrott, M. Diamond and K.J. Walsh (1954) Some factors affecting vehicular trafficability of snow. SIPRE Research Report 10. AD055056.

Hanamoto, B. (1971) Effect of snow cover on obstacle performance of vehicles. CRREL Technical Note (unpublished).

Hanamoto, B. (1974) Traction aid, Tyr-Trac in stamp sand. CRREL Technical Note (unpublished).

Hanamoto, B. (1975) Effects of variation in drawbar hitch location on vehicle performance. CRREL Special Report 237. AD016911.

Harrison, W.L. (1956) Preliminary study of snow values related to vehicle performance. Land Locomotion Research Laboratory Technical Memorandum.

Harrison, W.L. (1972) Snow surface strength as related to vehicle performance. CRREL Technical Note (unpublished).

Harrison, W.L. (1973) Vehicle traction on shallow snow cover. CRREL Technical Note (unpublished).

Harrison, W.L. (1975) Shallow snow performance of wheeled vehicles. CRREL Internal Report 438 (unpublished).

Harrison, W.L. (1975) Vehicle performance over snow: Math model validation study. CRREL Technical Report 268. ADA021228.

Harrison, W.L. and T. Czako (1961) Over-snow vehicle performance studies. Land Locomotion Research Laboratory Report No. LL-68.

Harrison, W.L., R. Berger and S. Takagi (1975) On the application of critical state soil mechanics to dry snow. CRREL Internal Report 473 (unpublished).

Hegedus, E. (1960) A simplified method for the determination of bulldozing resistance. Land Locomotion Research Laboratory Report No. LL-61.

Hettiaratchi, D.R.P. (1968) The calculation of passive earth pressure. Submitted for the degree of Doctor of Philosophy at the University of Newcastle upon Tyne. (unpublished).

Hodges, H.C. (1973) The role of tread design in skid resistance under winter driving conditions. Nevada Automotive Test Center, Carson City, Nevada.

Janosi, Z. and B. Hanamoto (1961) The analytical determination of drawbar pull as a function of slip for tracked vehicles in deformable soils. *Proceedings of the First International Conference on the Mechanics of Soil-Vehicle Systems,* Turin, Italy.

Johnson, P.R. (1979) A thickness/tensile stress/load relationship for ice. CRREL Technical Note (unpublished).

Kennedy, J.L. and E.L. Arriens (1972) Effects of snow parameters on vehicle mobility (partial report). U.S. Army Arctic Test Center, Seattle, Washington.

Kinoshita, S. and E. Akitaya (1970) Classification of snow and ice on roads. *Snow Removal and Ice Control Research, Proceedings of an International Symposium.* Highway Research Board, U.S. Department of Transportation.

Kovacs, A., F. Michitti and J. Kalafut (1977) Unconfined compression tests on snow. CRREL Special Report 77-20. ADA062445.

Landauer, J.K. (1955) Stress-strain relations in snow under

uniaxial compression. SIPRE Research Report 12. AD059687.

Landauer, J.K. and T. Royce (1956) Energy of snow compaction and its relation to trafficability. SIPRE Research Report 14. AD122666.

Liston, R.A. (1972) Articulated vehicle slope climbing ability with and without positive pitch control. CRREL Technical Note (unpublished).

Liston, R.A. (1972) Comments on the measurement of snow strength for prediction of vehicle performance. CRREL Technical Note (unpublished).

Liston, R.A. (1974a) Operation of the M35, 2½ ton truck in shallow snow. CRREL Technical Note (unpublished).

Liston, R.A. (1974b) Variation of tracked vehicle snow deformation patterns with load. CRREL Technical Note (unpublished).

Liston, R.A. and E. Hegedus (1964) Dimensional analysis of load sinkage relationships in soils and snow. U.S. Army Tank-Automotive Center Tech Report 8692.

Liston, R.A. (1977) An evaluation of vehicle traction aids for shallow snow operations. CRREL Technical Note (unpublished).

McGowan, R.P. and C.J. Roma (1961) Scaled vehicle mobility tractors (scale model tires in snow). U.S. Army Transportation Research Command, Fort Eustis, Virginia.

Mellor, M. (1963) Oversnow transport. CRREL Cold Regions Science and Engineering Monograph III-A4. AD404778.

Mellor, M. (1964) Properties of snow. CRREL Cold Regions Science and Engineering Monograph III-A1. AD611023.

Mellor, M. (1974) A review of basic snow mechanics. CRREL Internal Report 352 (unpublished).

National Research Council of Canada (1954) International classification for snow. Associate Committee on Soil and Snow Mechanics. Tech. Memo. 31: Ottawa, Canada.

Nevel, D.E. (1978) Bearing capacity of river ice for vehicles. CRREL Report 78-3. ADA055244.

Nuttall, C.J. (1957) On the sinkage and motion resistance of tracked vehicles in a shallow snow cover. Appendix A, Thomson, J.G. (1957).

Nuttall, C.J. and J.P. Finelli (1955) Vehicles in snow: A critical review of the state of the art. Tech Memo 3-414, Stevens Institute of Technology, Hoboken, New Jersey.

Nuttall, C.J. and R.P. McGowan (1961) Scale models of vehicles in soils and snows. U.S. Army Transportation Research Command, Fort Eustis, Virginia.

Nuttall, C.J. and J.G. Thomson (1958a) A study of some phases of the snow/vehicle interaction. Vol. I. Defence Research Board, Ottawa, Canada.

Nuttall, C.J. and J.G. Thomson (1958b) A study of some phases of the snow/vehicle interaction. Vol. II. Defence Research Board, Ottawa, Canada.

Nuttall, C.J., P.M. Jurkat and P.W. Haley (1975) The AMC '74 mobility model, USATACOM Technical Report No. 11921 (ADA-014278).

Pleuthner, R.L. (1969) A critique on the performance of off-road vehicles: Full scale test results and prediction method evaluation. Cornell Aeronautical Laboratory Inc., Buffalo, N.Y.

Reece, A.R. (1965) The fundamental equation of earthmoving mechanics. *Symposium on Earthmoving Machinery, Automotive Division, Institution of Mechanical Engineers*, vol. 179, part 3E.

Stephen, J.A. (1974) The relationship between drawbar pull and acceleration. CRREL Technical Note (unpublished).

Sugg, R.W. (1966) The drag of a large and small pneumatic tyre travelling through water, slush and snow. S&T Memo 9/66, Ministry of Aviation, U.K.

Thomson, J.G. (1957) A study of some factors influencing vehicle mobility in snow. Defence Research Board, Ottawa, Canada.

U.S. Army Engineer Waterways Experiment Station (1960) Tests on subarctic snow. Technical Memorandum No. 3-414, report 4.

U.S. Army Transportation Board (1962) Vehicle evaluations in subarctic environment. Fort Eustis, Virginia.

Yong, R.N. and W.L. Harrison (1978) On vehicle mobility in snow-covered terrain. *Journal of Terramechanics*, vol. 15, no. 4, p. 223-235.

☆ U.S. GOVERNMENT PRINTING OFFICE: 1981 500-259/204

A facsimile catalog card in Library of Congress MARC format is reproduced below.

Harrison, W.L.
Shallow snow model for predicting vehicle performance / by William L. Harrison. Hanover, N.H.: U.S. Cold Regions Research and Engineering Laboratory; Springfield, Va.: available from National Technical Information Service, 1981.
iv, 26 p., illus.; 28 cm. (CRREL Report 81-20.)
Prepared for Directorate of Military Programs - Office of the Chief of Engineers by Corps of Engineers, U.S. Army Cold Regions Research and Engineering Laboratory under DA Project 4A762730AT42.
Bibliography: p. 20.
1. Mobility. 2. Models. 3. Shallow depth. 4. Snow. 5. Snow vehicles. 6. Traction. I. United States. Army. Corps of Enginers. II. Army Cold Regions Research and Engineering Laboratory, Hanover, N.H. III. Series: CRREL Report 81-20.